MERCATOR'S NET.

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IN the belief that the subject of Map Projections has more value for and hold upon those who can follow the processes of construction or calculation from beginning to end, an attempt is made in this paper to bring the familiar, but often imperfectly grasped, Mercator's Net within the range of such mathematics as a student of Geography may ordinarily be supposed to possess. This does not include Integral Calculus. It is our purpose to set out the process in a simpler manner without resorting to anything that may afterwards have to be unlearnt, and, indeed, by means which will be helpful in higher branches.

The accompanying diagram brings out, in the first place, the contrast between Mercator's Net on the right and the Central Cylindrical Projection on the left. The two are still often confused. They are similar in appearance, since both have a rectangular mesh, and they are identical as regards the parallel straight lines which represent the Meridians. The difference arises only in the spacing of the parallel straight lines which represent the lines of Latitude. There is increase polewards in both cases, but the increase in Mercator's Net is not so rapid, the 70th parallel being about the same distance from the Equator as the 60th in the Central Cylindrical.

The construction of the Central Cylindrical is simple, lines being drawn from the centre through points on the sphere to a cylinder, usually tangential along the Equator. The quadrant to the left of the line CN shows the process in section for a 10° mesh, and may be allowed to explain itself.

Mercator's Net, on the other hand, cannot be constructed in any such simple geometrical way. Hence it is preferable not to call it a Projection. The quadrant to the right of the line may nevertheless be useful as indicating the principle on which the distances of the Parallels from one another, and thus from the Equator, are found. It is divided, as in the other case, by radii at intervals of 10°. From the points at which these intersect the circumference, lines are dropped parallel to the axis, to meet the radius next below. If now, with dividers, we transfer these distances (such as AP and BQ) successively to a meridian, we shall have the arrangement represented along the vertical line (springing from E) immediately to the right. This is Mercator's arrangement in principle, from which the true procedure only differs in that the intervals must be very small, in strict theory infinitesimal. The total of the distances will then become infinite, so that the Pole and its neighbourhood cannot be represented.

As an intermediate step, along the second vertical line to the right, is given the result which would be obtained by taking 1° intervals. This is not very different from the true Mercator up to about 60°, but the Parallels still fall markedly short of their true positions in higher Latitudes. It is, of course, not easily practicable in this case to arrive at the result geometrically, but the lengths such as BQ can be calculated. Example:—

$$BQ = BZ - QZ = CZ \tan 70° - CZ \tan 69°$$
$$= CZ (\tan 70° - \tan 69°)$$
$$= R \cos 70° (\tan 70° - \tan 69°)$$

where R is the radius, CB.

This is a somewhat arduous, though straightforward, task. It would be far more arduous for one minute intervals.

Another method of attack, however, presents itself here. For with such a small interval the radii CB and CA would be nearly parallel, so that CBA and CAB could be taken as right angles, and the arc AB as a straight line.

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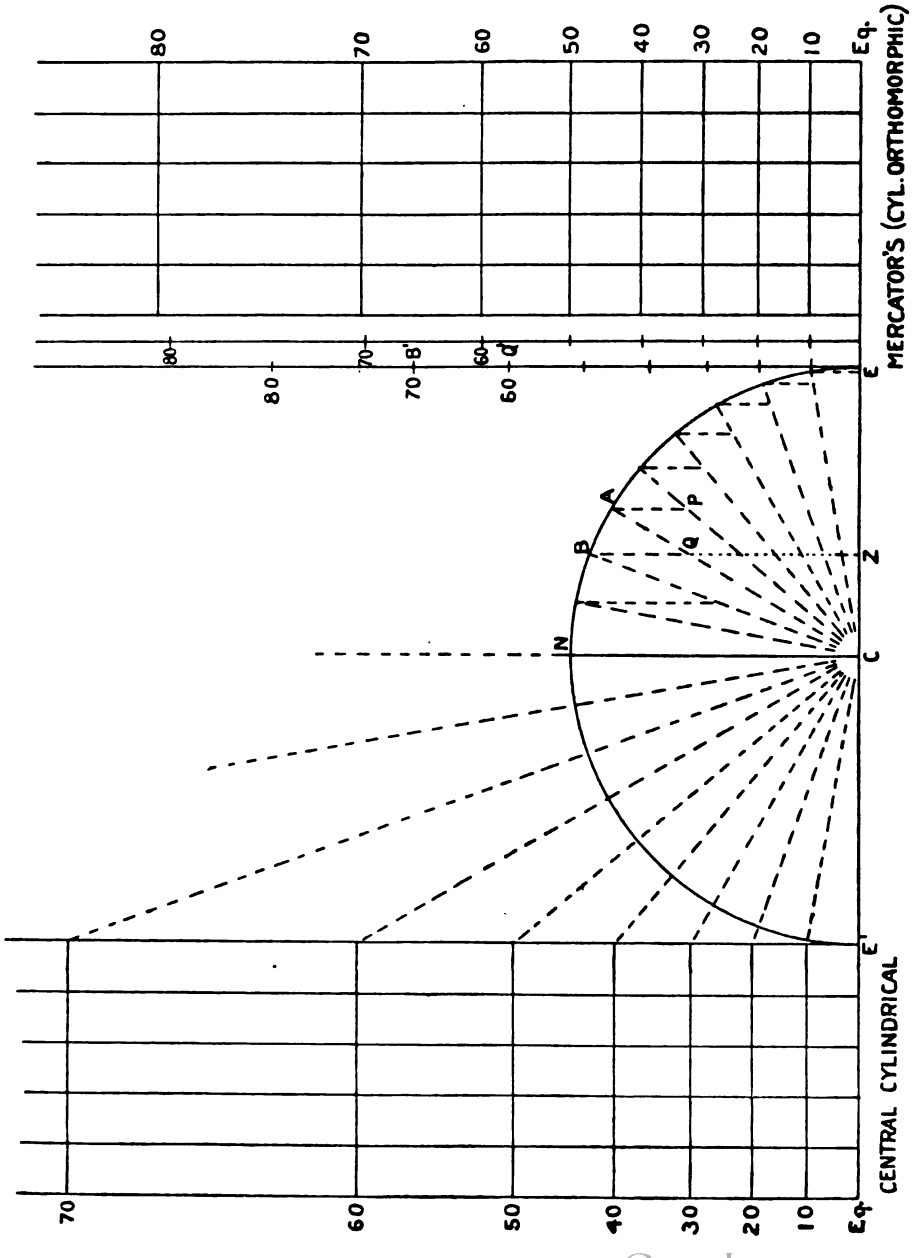
Then the angle ABQ would be complementary to CBQ; but BCZ is complementary to CB(Q)Z.

$$\therefore \quad ABQ = BCZ$$

$$\therefore \quad \frac{BQ}{BA} = \sec ABQ = \sec BCZ$$

This means that for *very small* intervals, the exaggeration of the distance between the Parallels is proportional to the secant of the Latitude. Thus a table of what are called Meridional Parts may be made by the



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addition of secant values for small intervals. Theoretically, as has been said, the interval ought to be infinitely small, but practically, as our figures shew, an interval of one minute gives a very good result, up to at least 85°, beyond which the Equatorial Mercator's Net is rarely if ever drawn.

We are still faced with the forbidding task of summing some 5000 secant values; but this may be simplified by taking the mean value of the secants for each degree, and multiplying by 60, which gives practically the same result as adding the secant values by minutes.

Thus even at such a high latitude as 69°—70° we have:

sec 70° = 2 924
sec 69° = 2·790

2|5·714
2·857 Mean secant value
60

171·420 Estimated total of secants by minutes

whereas actual addition by minutes in Chambers' Mathematical Tables gives 171·42 (4).

But a still greater saving of work may be made as follows:—

Suppose Parallels to be required at every 10°. Let m, n, o—u, v, w be the values of any 11 secants at every degree, beginning from one multiple of 10° and ending with another.

Then the sum of the mean values

$$= \frac{m+n}{2} + \frac{n+o}{2} + \cdots + \frac{u+v}{2} + \frac{v+w}{2}$$

$$= \frac{m}{2} + (n + o + \cdots + u + v) + \frac{w}{2}$$

Hence it is only necessary to take out the first and last of the set, divide their sum by 2, and add to this the sum of the intermediate values. The total multiplied by 60 will give a very close approximation to the sum of the secants by minutes. About half-an-hour's work in this way with a three-figure table gives the following results, which do not differ even in the units figure from the Meridional Parts in Chambers' Tables (pp. 418-426), except at 80°, where Chambers gives 8375·3.

Lat.	10°	20°	30°	40°	50°	60°	70°	80°
	603·1	1225·1	1888·4	2622·8	3474·5	4527·5	5966·5	8378·1

Now, assuming the Earth to be a sphere with a radius of 6,371,300 metres, and taking a scale of 1 : 100,000,000, we get 11·12 mm. as the scale length of 10° or 600′ along the Equator.

Thus the scale distances of the Parallels from the Equator work out proportionately as:—

Lat.	10°	20°	30°	40°	50°	60°	70°	80°
	11·18	22·71	35·00	48·61	64·39	83·91	110·58	155·27 mm

With Chambers' figure the last comes to 155.22 mm., a trifling difference. Even at 85° our method gives 199.72 mm., as against 199.51 mm. from the truer figure.

In order that a comparison may be made with figures based on the true spheroidal dimensions, we give a table taken from p. 166 of Karten-Entwurfslehre (Zöppritz-Bludau).

Lat.	10°	20°	30°	40°	50°	60°	70°	80°
	11·11	22·58	34·82	48·38	64·13	83·62	110·27	154·95

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A note on the term " orthomorphic " may usefully be added. On the globe, as is well known, the longitude distance for a given interval decreases in proportion to the cosine of the Latitude. If then the longitude distances are everywhere made equal on the map (as is necessarily the case where the Meridians are parallel straight lines, crossed at right angles by straight lines to represent the parallels), this must give an exaggeration in the opposite sense, i.e., proportional to the secant of the Latitude.

Thus at 60° on the globe the longitude distance is half that at the Equator. But with a rectangular net they are made equal. Hence the exaggeration at 60° is 2, the secant value. But on Mercator's Net, as has been shewn, the latitude distances for very small intervals are also exaggerated as the secant of the Latitude.

Thus for very small distances round any point, the scale, though it may be too large, will be exaggerated in the same proportion in both directions, or, what comes to the same thing, the angles immediately round the point will remain true. This is called orthomorphism, a property sometimes said to be more interesting mathematically than useful practically. But in Mercator's Net we have an application of great practical value. For if a line (a loxodrome or rhumb-line) be drawn on the globe so as to cut successive Meridians at a constant angle, and if this line be then transferred to a Mercator Chart, it must come out as a straight line, since the orthomorphism of the network keeps the angles true in detail, and a line which cuts a close succession of parallel straight lines at a constant angle can only be a straight line.

Conversely, a straight line between two points on a Mercator Chart, represents a constant (if not the shortest) course, and this can be approximately followed with a Compass. Hence its usefulness in Navigation.

Maps and Survey. By Arthur R. Hinks. 5¾in. x 8¾in. pp. 258. 12/6 Cambridge Press.

It is ten years ago since the first edition of this now well-known book appeared, and in the meantime each of the subjects included in the title has been the object of much experiment, and a large amount of such experimental work has been done by Mr. Hinks himself. The war gave a great stimulus to the production of new and better maps, and solely on account of that phase of surveying and map production it has become necessary to supplement the first edition. We gather that the present book is a temporary expedient until such time as the whole can be re-written. The first 201 pages have been re-printed without alteration. Certain additions and corrections—none of the latter is serious—have been collected into a new chapter. Probably the most important addition here is an alternative method of resection with the plane table, which gives a direct solution. The new developments in connection with levelling, as set out in lines 1 and 2, p. 207, already seem to need some bringing up to date, as the work is rather further advanced than the paragraph seems to suggest (see Geographical Teacher, Summer, 1921). It is true that the entire set of new levels will not be marked on the O.S. maps for some little time. Chapter X, " A further Chapter on Maps," is entirely new, and collects together a great mass of most interesting items, such as: Flying Maps, Conventional Signs, Map Reproduction, Work of P.C.G.N (see end pages of Geographical Teacher, Autumn, 1922, and Page 1, Spring, 1923), and Map Analysis. The value of the last section is admitted, but the author himself suggests that a single sheet of a series may or may not be representative of the value of the whole series, and as over 30 more or less different series are analysed, it is a great pity that " circumstances have not allowed the reproduction of any new maps " in the second edition. Some of those analysed are not easily obtained, and an analysis is of much diminished value in the absence of the map concerned. Chapter XI, " Maps and Survey in War," deals with The Grid, Survey for Artillery, Map Supply during the War, Sound Ranging, Flash Spotting, and Enemy Survey. Much has already been published under the several heads, but Mr. Hinks has carefully summarised and correlated the available material. It is a most attractively written chapter, and will appeal to a very wide circle. Chapter XII, " New Methods of Survey," is very much more technical, as it sets out all the recent work on Photographic and Stereoscopic Surveys. Some of this work has been published in the " Geographical Journal," particularly with regard to the stereo-autograph of Von Orel and Zeiss, but its inclusion in the present volume is a distinct advantage in a text book which will rank as the standard summary of all branches of survey and map work. W.E.W.

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